

Likelihood and Bayesian estimation of $P(Y < X)$ using lower record values from a general class of distributions

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Abstract In this paper, we study inference for the stress-strength reliability based on lower record data, where the stress and the strength variables are modeled by two independent but not identically distributed random variables from distributions belonging to the proportional hazard family. Likelihood and Bayesian estimators are derived and the confidence intervals and credible sets, respectively, are obtained. Finally, some numerical results are reported in order to show the performance of the proposed procedures.

Abstract *Nel presente lavoro si analizzano alcuni aspetti inferenziali relativi alla misura dello stress-strength basata su dati di tipo lower record, ipotizzando che stress e strength possano essere descritte da due variabili casuali indipendenti, ma non identicamente distribuite, appartenenti ad una data classe generale di distribuzioni. Vengono derivati stimatori di massima verosimiglianza e Bayesiani, ottenendo per essi rispettivamente, gli intervalli di confidenza e di credibilità. Infine, vengono riportati alcuni risultati numerici al fine di mostrare la performance delle procedure proposte.*

Key words: reliability, stress-strength model, confidence interval, credible set, Topp-Leone Distribution

1 Introduction

In reliability studies, the stress-strength term is often used to describe the life of a component which has a random strength X and is subject to a random stress Y . The quantity $\Phi = P(Y < X)$ is a measure of a component reliability which has many applications in physics, engineering, genetics, psychology and economics [4].

Let $X_1, X_2, \dots$ be an infinite sequence of independent and identically distributed random variables from an absolutely continuous distribution function (df) F , and probability density function (pdf) f . An observation X_j is called a lower record value if its value is less than that of all previous observations, i.e. X_j is a lower record if $X_j < X_i$ for all $i < j$. The record time sequence is defined as $T_1 = 1$ and $T_n = \min \{j : X_j < X_{T_{n-1}}\}$, for $n > 1$. The lower record value sequence $R_1, R_2, \dots, R_n$ is defined as $R_n = X_{T_n}$, $n = 1, 2, \dots$. The pdf of R_n is given by [2].

$$f_{R_n}(r_n) = \frac{1}{\Gamma(n)} [-\ln F(r_n)]^{n-1} f(r_n). \quad (1)$$

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Moreover, the joint *pdf* of the set of records $R_1, R_2, \dots, R_n$ is

$$f_{R_1, \dots, R_n}(r_1, \dots, r_n) = f(r_n) \prod_{i=1}^{n-1} rh(r_i). \quad (2)$$

with $r_1 > r_2 > \dots > r_n$, where $rh(r_i) = \frac{f(r_i)}{F(r_i)}$ is the reversed hazard rate (*rhr*).

Recently, some authors (see for example [5]) have placed considerable interest in flexible families of distributions defined as

$$F_Z(z) = [G(z)]^\theta, \quad (3)$$

with $-\infty \leq a < z < b \leq \infty$ and $\theta > 0$, where $G(\cdot)$ is an arbitrary continuous *df* free of unknown parameter and $G(a) = 0$, $G(b) = 1$. We refer to $G(\cdot)$ as the generating *df* or baseline distribution.

In literature the *df* defined in (3) is called with different names such as: proportional reversed hazard family, Lehmann alternatives, power distribution. The distribution (3) includes several well-known distribution as special cases: generalized exponential distribution introduced by [3] $F(x) = (1 - e^{-x})^\theta$, the Burr X distribution $F(x) = (1 - e^{-x^2})^\theta$, the Burr III distribution $F(x) = (1 + x^{-\gamma})^{-\beta}$ with $\gamma = \gamma_0$ known, the Topp-Leone distribution $F(x) = (2x - x^2)^\gamma$ with $x \in (0, 1)$.

The *pdf* and *rhr* corresponding to (3), respectively, are

$$f_Z(z; \theta) = \theta [G(z)]^{\theta-1} g(z) \quad \text{and} \quad rh_Z(z; \theta) = \theta rh_G(z),$$

where $g(\cdot)$ and $rh_G(\cdot)$ are the *pdf* and *rhr* corresponding to baseline distribution $G(\cdot)$, respectively. In this work, we suppose that the stress (Y) and strength (X) are two independent but not identically distributed random variables with *df* defined in (3). In particular, we assume that

$$X \sim F_X(x; \theta_1) = [G(x)]^{\theta_1} \quad \text{and} \quad Y \sim F_Y(y; \theta_2) = [G(y)]^{\theta_2},$$

where the symbol “ $\sim$ ” indicates “distributed as”. In this case, one can simply verify that the stress-strength parameter is given by

$$\Phi = P\{Y < X\} = \theta_1 \int_{\mathfrak{R}} [G(x)]^{\theta_1 + \theta_2 - 1} dG(x) = \frac{\theta_1}{\theta_1 + \theta_2}. \quad (4)$$

The main aim of this paper is to develop different inferential procedures for the stress-strength reliability Φ , when X and Y are two independent but not identically distributed random variables with *df* defined in (3), based on lower record data.

The paper is organized as follows. In Section 2, we will provide the distribution of maximum likelihood (*ML*) of the parameter θ of the distribution defined in (3) based on the lower record data. Confidence intervals based on the exact as well as asymptotic distribution of the *ML* estimator (*MLE*) of Φ are derived. In Section 3, we will discuss Bayesian inference on Φ under assumption of independent gamma as well as Jeffreys' priors. In Section 4, we will study the important special case of proportional reversed hazard family defined in (3), namely the Topp-Leone distribution. Finally, using simulation studies, different methods of estimation of Φ are compared.

2 Likelihood inference

In this section, first of all, we determine the distribution of *MLE* of θ based on the lower record data. Successively, using the distribution of *MLE* of θ , we derive the exact distribution of *MLE* of Φ . To this end, let $\mathbf{r} = (r_1, \dots, r_m)$ be the lower record values from *df* (3). The likelihood function is

$$L(\theta; \mathbf{r}) = \theta^m [G(r_m)]^\theta \prod_{i=1}^m r h_G(r_i). \quad (5)$$

The *MLE* of θ is given by

$$\hat{\theta}_{ML} = \frac{m}{-\ln[G(r_m)]} \quad (6)$$

and the Fisher information is $I(\theta) = \frac{m}{\theta^2}$. It is worthwhile pointing out that the *MLE* of θ depends only on the m -th lower record value r_m . The following Proposition provides the distribution of *MLE* of θ .

Proposition 1 *The MLE $\hat{\theta}_{ML}$ of the parameter θ of distribution function defined in (3) has an Inverse Gamma distribution with parameters m and $m\theta$, i.e.*

$$\hat{\theta}_{ML} \sim IG(m, m\theta) \quad (7)$$

Proof. By (1) and (3), the *pdf* of R_m is: $f_{R_m}(r_m) = \frac{\theta[G(r_m)]^{\theta-1} g(r_m) [-\ln\{G(r_m)\}]^{\theta-1}}{\Gamma(m)}$. Let $Y = -\ln G(R_m)$ be the random variable (*rv*) with inverse given by $R_m = G^{-1}(e^{-Y})$ and $\frac{\partial R_m}{\partial Y} = \frac{-e^{-Y}}{g[G^{-1}(e^{-Y})]}$. Using standard transformation techniques of *rv*, it is simple to verify that Y is Gamma distributed with *pdf* given by $f_Y(y) = f_{R_m}(G^{-1}(e^{-Y})) \frac{e^{-Y}}{g[G^{-1}(e^{-Y})]} = \frac{\theta^m}{\Gamma(m)} y^{m-1} e^{-\theta y}$. Consequently, the *pdf* of $\hat{\theta}_{ML} = \frac{m}{Y}$ is $f_{\hat{\theta}_{ML}}(\hat{\theta}_{ML}) = \frac{(m\theta)^m}{\Gamma(m)} \hat{\theta}_{ML}^{-m-1} \exp\left\{-\frac{m\theta}{\hat{\theta}_{ML}}\right\}$ which is the Inverse Gamma distribution with parameters m and $m\theta$.

From general properties of Inverse Gamma distribution, we can say that the *MLE* $\hat{\theta}_{ML}$ is a biased estimator of θ . In fact, we have $E[\hat{\theta}_{ML}] = \frac{m}{m-1}\theta$ for $m > 1$. Moreover, for $m > 2$, the variance of $\hat{\theta}_{ML}$ is $V[\hat{\theta}_{ML}] = \frac{m^2}{(m-1)^2(m-2)}\theta^2$. Trivially, an unbiased estimator for θ is $\hat{\theta}_{ML,U} = \frac{m-1}{m}\hat{\theta}_{ML}$, with variance given by $V[\hat{\theta}_{ML,U}] = \frac{\theta^2}{(m-2)}$, for $m > 2$.

In the final part of this section, we now discuss the distribution of the *MLE* of Φ using lower record data. For this purpose, let us suppose that $\mathbf{r} = (r_1, \dots, r_m)$ and $\mathbf{s} = (s_1, \dots, s_n)$ are independent sets of lower records from $F_X(x; \theta_1) = [G(x)]^{\theta_1}$ and $F_Y(y; \theta_2) = [G(y)]^{\theta_2}$, respectively. By Proposition 1, we have $\hat{\theta}_{1,ML} \sim IG(m, m\theta_1)$ and $\hat{\theta}_{2,ML} \sim IG(n, n\theta_2)$. Hence, using the equivariance property of *MLE*, the *MLE* $\hat{\Phi}_{ML}$ of Φ can be obtained by substituting $\hat{\theta}_{j,ML}$ in place of θ_j in (4) for $j = 1, 2$. That is, the *MLE* of Φ is given by

$$\hat{\Phi}_{ML} = \frac{\hat{\theta}_{1,ML}}{\hat{\theta}_{1,ML} + \hat{\theta}_{2,ML}}. \quad (8)$$

The following Proposition provides the distribution of *MLE* $\hat{\Phi}_{ML}$ of Φ .

Proposition 2 *The pdf of MLE $\hat{\Phi}_{ML}$ of Φ is*

$$f_{\hat{\Phi}_{ML}}(\hat{\Phi}_{ML}) = \frac{m^m n^{-m}}{B(n, m)} \left(\frac{\theta_2}{\theta_1}\right)^m \frac{(1 - \hat{\Phi}_{ML})^{m-1}}{(\hat{\Phi}_{ML})^{m+1}} \left\{1 + \frac{m\theta_2(1 - \hat{\Phi}_{ML})}{n\theta_1 \hat{\Phi}_{ML}}\right\}^{-n-m}. \quad (9)$$

Proof. In order to determine the *pdf* of $\hat{\Phi}_{ML}$, we consider the following *rv* $W_1 = \frac{2m\theta_1}{\hat{\theta}_{1,ML}}$. By Proposition 1, it is simple to verify that $W_1 \sim \chi^2(2m)$. Similarly, $W_2 = \frac{2n\theta_2}{\hat{\theta}_{2,ML}} \sim \chi^2(2n)$. Hence, given that W_1 and W_2 are independent, we have that $W = \frac{2nW_1}{2mW_2} \sim F_{2m, 2n}$ where $F(\cdot, \cdot)$ is the well-known F-distribution, with density function given by $f_W(w) = \frac{m^m n^n}{B(m, n)} \frac{w^{m-1}}{(n+mw)^{m+n}}$. After observing that $\hat{\Phi}_{ML}$ can be rewritten as $\hat{\Phi}_{ML} = \left\{1 + \frac{\theta_2}{\theta_1} W\right\}^{-1}$, using transformation techniques of random variable, by $f_W(w)$ we get (9).

By now, we discuss the exact and approximate interval estimation for Φ based on the lower record data. In order to construct an exact confidence interval, we observe that the *rv* W , after simple algebra, can be rewritten as $W = \frac{\Phi}{1-\Phi} \frac{\hat{\theta}_{2,ML}}{\hat{\theta}_{1,ML}}$. Now, given that W has an F distribution with $2m$ and $2n$ degrees of freedom,

W is a pivotal quantity for Φ . Therefore, having fixed $\gamma_1, \gamma_2 \in (0, 1)$ such as $\gamma_1 + \gamma_2 = \gamma$, with $\gamma \in (0, 1)$, observed r_m and s_n , calculated the *MLE* $\hat{\theta}_{1,ML}$ and $\hat{\theta}_{2,ML}$, an exact $100(1 - \gamma)\%$ confidence interval for Φ is:

$$\left\{ \left(\frac{\hat{\theta}_{2,ML}}{w_{\gamma_1} \hat{\theta}_{1,ML}} + 1 \right)^{-1}, \left(\frac{\hat{\theta}_{2,ML}}{w_{\gamma_2} \hat{\theta}_{1,ML}} + 1 \right)^{-1} \right\}, \quad (10)$$

where w_{γ_1} and w_{γ_2} are such that $P[W < w_{\gamma_1}] = \gamma_1$ and $P[W < w_{\gamma_2}] = 1 - \gamma_2$, respectively.

Intervals based on the asymptotic normality of *MLE* can be of interest in cases when the number of records is sufficiently large. Following [1], let $p_1 = \lim_{n,m \rightarrow \infty} \frac{m}{n+m}$ and $p_2 = \lim_{n,m \rightarrow \infty} \frac{n}{n+m}$, the entries of

the expected Fisher information matrix $\mathbf{I}(\theta_1, \theta_2)$ are $I_{ii} = \lim_{n,m \rightarrow \infty} E \left[-\frac{1}{m+n} \frac{\partial^2 \ell(\theta_1, \theta_2)}{\partial \theta_i^2} \right] = \frac{p_i}{\theta_i^2}$, for $i = 1, 2$,

and $I_{12} = I_{21} = 0$. Under regularity conditions, the asymptotic distribution of *MLE* $\hat{\Phi}_{ML}$ of Φ , satisfies $\sqrt{m+n}(\hat{\Phi}_{ML} - \Phi) \xrightarrow{d} N(0, V^2)$, where asymptotic variance is $V^2 = \mathbf{g}[\mathbf{I}(\theta_1, \theta_2)]^{-1} \mathbf{g}' = \Phi^2(1 - \Phi)^2 \left(\frac{1}{p_1} + \frac{1}{p_2} \right)$ with $\mathbf{g} = \left[\frac{\partial \Phi}{\partial \theta_1}, \frac{\partial \Phi}{\partial \theta_2} \right]$. A $100(1 - \gamma)\%$ approximate confidence interval for Φ is

$$\left\{ \hat{\Phi}_{ML} \pm z_{\frac{\gamma}{2}} \frac{\hat{V}}{\sqrt{n+m}} \right\}, \quad (11)$$

where $\hat{V}$ is obtained by substituting $\frac{m}{n+m}$ for p_1 , $\frac{n}{n+m}$ for p_2 , and *MLE* $\hat{\theta}_{1,ML}$ and $\hat{\theta}_{2,ML}$ in the asymptotic standard deviation V .

3 Bayesian inference

In this section, we discuss the Bayesian inference of Φ based on the derivation of the posterior *pdf* of Φ which can be obtained by using a suitable one-to-one transformation of random variables θ_1 and θ_2 .

By looking at the likelihood function of θ_i given the set of lower record values, we elicit that θ_i has a conjugate gamma prior distribution with parameters δ_i and β_i , so that the posterior density is a Gamma distribution *i.e.* $\theta_i | \mathbf{t} \sim G(\delta_i + l, [\beta_i - \ln G(t_l)])$ where $\mathbf{t} = \mathbf{r}$ or $\mathbf{t} = \mathbf{s}$, $l = m$ or $l = n$ and $t_l = r_m$ or $t_l = s_n$. Given that under a squared error loss function the Bayes estimator of θ_i is the mean of the posterior density, we have

$$\hat{\theta}_{i,B} = E[\theta_i | \mathbf{t}] = \frac{\delta_i + l}{[\beta_i - \ln G(t_l)]}. \quad (12)$$

Since the random variables θ_1 and θ_2 are independent, their joint posterior density is

$$\varphi(\theta_1, \theta_2 | \mathbf{r}, \mathbf{s}) = \frac{a_1^{\delta_1+m} \theta_1^{\delta_1+m-1} e^{-\theta_1 a_1}}{\Gamma(\delta_1+m)} \frac{a_2^{\delta_2+n} \theta_2^{\delta_2+n-1} e^{-\theta_2 a_2}}{\Gamma(\delta_2+n)}. \quad (13)$$

where $a_1 = [\beta_1 - \ln G(r_m)]$ and $a_2 = [\beta_2 - \ln G(s_n)]$. Following [4] and [7], we obtain the posterior *pdf* of Φ by mean of a one-to-one transformation of the type $H : (\theta_1, \theta_2) \rightarrow (\Phi, \lambda)$ with inverse $Q = H^{-1}$. So, putting $\Phi = \frac{\theta_1}{\theta_1 + \theta_2}$ and $\lambda = \theta_1 + \theta_2$, taking into account that the Jacobian of transformation is λ , by (13) the joint *pdf* of (Φ, λ) is:

$$\pi(\Phi, \lambda | \mathbf{r}, \mathbf{s}) = \frac{a_1^{\delta_1+m} a_2^{\delta_2+n} \Phi^{\delta_1+m-1} (1-\Phi)^{\delta_2+n-1}}{\Gamma(\delta_1+m) \Gamma(\delta_2+n)} \lambda^{\delta_1+\delta_2+m+n-1} e^{-\lambda \{ \Phi a_1 + (1-\Phi) a_2 \}}. \quad (14)$$

Consequently, we can obtain the posterior *pdf* of Φ marginalizing (14) with respect to λ ; that is

$$\pi_{\Phi}(\Phi|r, s) = \int_0^{\infty} \pi(\Phi, \lambda|r, s) d\lambda = \frac{a_1^{\delta_1+m} a_2^{\delta_2+n}}{B(\delta_1+m, \delta_2+n)} \frac{\Phi^{\delta_1+m-1} (1-\Phi)^{\delta_2+n-1}}{\{\Phi a_1 + (1-\Phi) a_2\}^{\delta_1+\delta_2+m+n}}. \quad (15)$$

Remark 3 It is simple to verify that under Jeffreys' prior $\pi(\theta_1, \theta_2) \propto \theta_1^{-1} \theta_2^{-1}$ the posterior pdf of Φ is:

$$\pi_{\Phi}^J(\Phi|r, s) = \frac{[-\ln G(r_m)]^m [-\ln G(s_n)]^n \Phi^{m-1} (1-\Phi)^{n-1}}{B(m, n) \{\Phi [-\ln G(r_m)] + (1-\Phi) [-\ln G(s_n)]\}^{m+n}}. \quad (16)$$

Note that $\pi_{\Phi}^J(\Phi|r, s)$ can be obtained by (15) putting $\delta_1 = \delta_2 = \beta_1 = \beta_2 = 0$.

After some algebra, the ξ -th moment of Φ is:

$$E[\Phi^{\xi}|r, s] = \left(\frac{a_1}{a_2}\right)^{\delta_1+m} \frac{B(\xi + p_1, p_2)}{B(p_1, p_2)} {}_2F_1\left(p_1 + p_2, \xi + p_1; \xi + p_1 + p_2; \frac{a_2 - a_1}{a_2}\right),$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric function, with $p_1 = \delta_1 + m$ and $p_2 = \delta_2 + n$. Consequently, the posterior mean is:

$$\hat{\Phi}_B = E[\Phi|r, s] = \left(\frac{a_1}{a_2}\right)^{p_1} \frac{p_1}{(p_1 + p_2)} {}_2F_1\left(p_1 + p_2, p_1 + 1; p_1 + p_2 + 1; 1 - \frac{a_1}{a_2}\right). \quad (17)$$

Under the Jeffreys' the posterior mean is:

$$\hat{R}_B^{(J)} = \left(\frac{-\ln G(r_m)}{-\ln G(s_n)}\right)^m \frac{m}{m+n} {}_2F_1\left(m+n, m+1; m+n+1; 1 - \frac{-\ln G(r_m)}{-\ln G(s_n)}\right). \quad (18)$$

From (15), an equi-tailed $100(1-\alpha)\%$ Bayesian credible set for Φ is $(\Phi_{(\alpha/2)}, \Phi_{(1-\alpha/2)})$ such that

$$\int_0^{\Phi_{(\alpha/2)}} \pi_{\Phi}(\Phi|r, s) d\Phi = \alpha/2 \quad \text{and} \quad \int_0^{\Phi_{(1-\alpha/2)}} \pi_{\Phi}(\Phi|r, s) d\Phi = 1 - \alpha/2.$$

It is also possible to obtain the highest posterior density (HPD) interval through numerical optimization techniques.

4 Example and Simulation: Topp-Leone distribution

We now consider two independent sets of lower records, $\mathbf{r} = (r_1, \dots, r_m)$ and $\mathbf{s} = (s_1, \dots, s_n)$, from two different Topp-Leone (TL) [6] random variables. A random variable X is said to follow a TL distribution if it has a *cdf* and a *pdf* respectively given by:

$$F_X(x; \nu) = (2x - x^2)^{\nu} \quad \text{and} \quad f_X(x; \nu) = 2\nu(1-x)(2x - x^2)^{\nu-1}, \quad (19)$$

with $\nu > 0$ and $0 \leq x \leq 1$. From (6) and (8), the MLE of ν_1 , ν_2 and Φ are given by:

$$\hat{\nu}_{1ML}(r_m) = \frac{m}{-\ln(2r_m - r_m^2)}, \quad \hat{\nu}_{2ML}(s_n) = \frac{n}{-\ln(2s_n - s_n^2)} \quad \text{and} \quad \hat{\Phi}_{ML} = \frac{\hat{\nu}_{1ML}(r_m)}{\hat{\nu}_{1ML}(r_m) + \hat{\nu}_{2ML}(s_n)},$$

and from (10) and (11), it is possible to obtain the exact and asymptotic confidence interval. Furthermore, from (16) and (18), we obtain the posterior density and the Bayes estimator of Φ under Jeffreys' prior.

In order to investigate the behavior of the proposed estimators, a simulation study is carried out. For fixed ν_1, ν_2 and sample size N , we generate $k = 1000$ independent samples from $G(x; \nu_1)$ and $G(y; \nu_2)$ and

compute the ML and Bayes estimate of Φ . Moreover, putting $\alpha = 0.05$, we compute the lower (LB) and upper bound (UB) for the exact and the asymptotic confidence interval and for the HPD interval for each of the k samples. The mean of the obtained estimates over all k samples, the empirical bias, the empirical mean square error (MSE) and the posterior risk ($PR = Var(\hat{\Phi}_B^{(j)} | \mathbf{r}, \mathbf{s})$) for Bayes estimator are shown in Table 1.

Table 1 Simulation results.

v_1	v_2	N	Φ	$\hat{\Phi}$	bias	MSE	$\hat{\Phi}_B^{(j)}$	$PR(\hat{\Phi}_B^{(j)})$	Exact CI			Asymptotic CI			HPD		
									LB	UB	Cov	LB	UB	Cov	LB	UB	Cov
0.5	0.8	10	0.385	0.468	0.083	0.036	0.472	0.035	0.145	0.820	0.952	0.083	0.853	0.916	0.186	0.783	0.787
		200	0.385	0.415	0.030	0.018	0.421	0.018	0.180	0.695	0.957	0.141	0.688	0.941	0.145	0.723	0.926
		1000	0.385	0.407	0.022	0.015	0.412	0.016	0.198	0.656	0.942	0.168	0.645	0.93	0.147	0.698	0.933
	2	10	0.2	0.412	0.212	0.067	0.426	0.072	0.117	0.790	0.853	0.030	0.794	0.907	0.141	0.756	0.727
		200	0.2	0.361	0.161	0.039	0.372	0.043	0.147	0.648	0.781	0.098	0.624	0.867	0.115	0.656	0.85
		1000	0.2	0.356	0.156	0.036	0.365	0.039	0.164	0.609	0.726	0.126	0.587	0.827	0.127	0.625	0.83
	3	10	0.789	0.753	-0.037	0.022	0.722	0.027	0.370	0.941	0.949	0.464	1.042	0.901	0.437	0.927	0.809
		200	0.789	0.773	-0.017	0.011	0.756	0.013	0.510	0.917	0.939	0.572	0.974	0.917	0.527	0.939	0.915
		1000	0.789	0.776	-0.014	0.009	0.762	0.010	0.547	0.908	0.945	0.598	0.953	0.913	0.555	0.936	0.934
	2	10	0.6	0.590	-0.010	0.030	0.577	0.028	0.220	0.882	0.96	0.219	0.961	0.896	0.284	0.855	0.804
		200	0.6	0.596	-0.004	0.019	0.589	0.019	0.317	0.824	0.951	0.328	0.865	0.914	0.278	0.875	0.907
		1000	0.6	0.590	-0.010	0.016	0.584	0.016	0.340	0.799	0.938	0.350	0.830	0.924	0.274	0.870	0.917

5 Conclusions

In this paper we deal with the problem of estimating the stress-strength reliability measure Φ based on two independent but not identically distributed random variables from a general class of distributions. Moreover, we consider different inferential procedures based on lower record data. We show that it is possible to obtain the *MLE* for Φ and its distribution, so to have both the exact and the asymptotic confidence interval. Next, in a Bayesian context, under gamma and Jeffreys' priors, we obtain the posterior density for Φ and the corresponding Bayes estimator. Finally, we consider two independent sets of lower record from Topp-Leone random variables and provide some numerical results. The results suggest a good performance of the considered procedures for finite sample sizes as well.

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